WISE: WAVELET IMAGE SPECTRA ENHANCEMENT OF X-RAY IMAGES

Dr. Bill Cardoso, *IEEE Senior Member*, Dr. Glen Thomas
Creative Electron, Inc.
San Marcos | Santa Cruz | Chicago

Abstract – The Wavelet Image Spectra Enhancement (WISE) technique was developed as a set of powerful filters designed to improve the quality of x-ray images. In this paper we describe how the transform thresholding and parameter estimation techniques in WISE are used as building blocks for radiographic feature augmentation. Transform thresholding techniques are applied to select the frequency coefficients of the discrete wavelet transform (DWT), the discrete cosine transform (DCT), and the discrete Fourier transform (DFT). Furthermore, a novel algorithm using the continuous wavelet transform (CWT) is also presented as a parameter estimation approach to filter x-ray imaging data. The data filtering performance of these algorithms is examined using both simulated and experimental x-ray signals. The results show that the parameter estimation has superior filtering performance when compared to the conventional transform thresholding techniques.

I. INTRODUCTION

The analysis, storage, and transmission of x-ray data can benefit from sharpening and noise suppression algorithms. The data augmentation performance of the DWT, the DFT, and the DCT are examined when applied to simulated and experimental x-ray data. An adaptive threshold is applied to remove the smaller coefficients of the frequency domains to achieve data filtering. Then, the signals are reconstructed from the remaining coefficients. The mean square errors of these techniques are analyzed for filtering and compression ratios of up to 90%. For higher ratios, the CWT is also investigated in this study. The CWT performs the correlation of a kernel wavelet with the radiographic signal. A modified version of the Morlet wavelet is used as the kernel wavelet to estimate the signal parameters. Upon the determination of these parameters, high data improvement and denoising can be obtained. The CWT is applied to both simulated and experimental data, and the results are presented in this paper.

II. THRESHOLDING TECHNIQUES

Subband transform (DWT) and transform coding techniques (DCT and DFT) provide a representation of the input signal into separate frequency bands. Data filtering is therefore achieved by selecting portions of the frequency domain where the signal is expected. Energy in the frequency domain outside the bandwidth of the signal is usually due to noise. This method is most effective for bandwidth limited signals with uncorrelated noise.

A. DWT, DFT, and DCT

The filter bank representation of the DWT [1] is shown in Figure 1. The lowpass $h_0(n)$ and the highpass $h_1(n)$ filters are analysis filters, while $g_0(n)$ and $g_1(n)$ are lowpass and highpass synthesis filters.

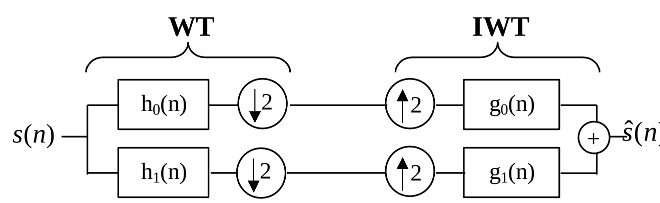


Figure 1 – Discrete Wavelet Transform

The analysis filters $h_0(n)$ and $h_1(n)$ can be selected as quadrature mirror filters (QMF) [2]. In such configuration $g_0(n)$ and $g_1(n)$ are the time reversed versions of $h_1(n)$ and $h_0(n)$ respectively. In order to recover the input signal from the DWT coefficients, $h_0(n)$ and $h_1(n)$ have to satisfy the following condition:

$$\sum h_i(n)h_i(n-2m) = \delta(m), \qquad i = 0,1 \qquad (1)$$

The DFT of a signal $f(n)$ (or f_N in a vector representation) of length N is defined in Equation (2). The frequency domain signal F_N is a linear combination of the time domain signal f_N with the kernel W_N.

$$F_N = W_N . f_N \qquad (2)$$

In the above equation W_N is an N×N symmetric matrix defined as:

$$W(k,n) = \exp\left(\frac{-j2\pi kn}{N}\right), \quad k = 0,1,\ldots,N-1, n = 0,1,\ldots,N-1$$

The inverse DFT (IDFT) is shown in Equation (3).

$$f_N = \frac{1}{N} W_N^{-1} . F_N \qquad (3)$$

The DCT and its inverse (IDCT) are defined in Equations (4) and (5) respectively.

$$F_N = C_N . f_N \qquad (4)$$

$$f_N = C_N^{-1} . F_N \qquad (5)$$

$$C(k,n) = \sqrt{\frac{2}{N}} \cos\left(\frac{\pi}{N}(k+0.5)(n+0.5)\right), \quad k = 0,1,\ldots,N-1, n = 0,1,\ldots,N-1$$

Data filtering of a given signal *f(n)* is successful when the redundant and noise components of *f(n)* are reduced or removed in the frequency domain (Figure 2). The signal $\hat{f}(n)$ is the filtered representation of *f(n)*. The following section describes how thresholding for data compression and noise suppression can be applied to the frequency coefficients of the original radiographic signal.

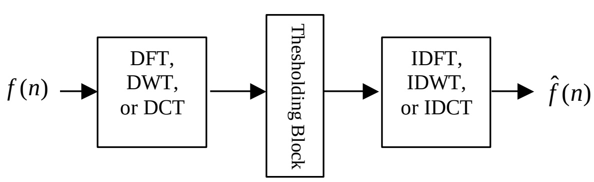


Figure 2: Thresholding of frequency coefficients

B. *Adaptive Threshold*

For the case of a signal corrupted by additive white Gaussian noise (AWGN) with variance σ^2, reference [3] has shown that the optimal threshold is given by:

$$\tau = \frac{\sigma}{\sqrt{N}} \sqrt{2.\ln(N-1)} \qquad (6)$$

This "universal" threshold τ is applied to radiographic data corrupted by WGN. In the threshold approach all transform coefficients smaller than τ are set to zero. All coefficients greater than τ are kept the same:

$$\hat{W}_f(n) = \begin{cases} 0, & |W_f(n)| < \tau \\ W_f(n), & |W_f(n)| \geq \tau \end{cases}, n = 0 \ldots N-1 \qquad (7)$$

This approach is referred to as a hard thresholding technique [4]. In the above equation $W_f(n)$ represents the frequency coefficients of the radiographic signal *f(n)*, while $\hat{W}_f(n)$ is the set of frequency coefficients used in the IDWT, IDCT, and IDFT. Figure 3 shows the input/output mapping for the hard thresholding case.

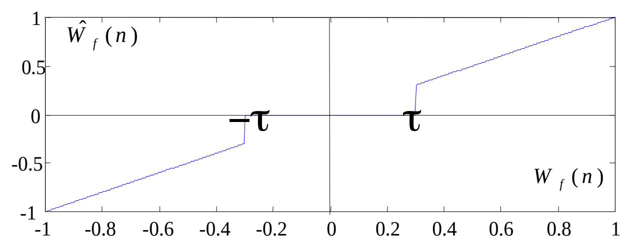


Figure 3: Hard thresholding rule.

III. Parameter Estimation Technique

The parameter estimation method is based on the CWT to estimate the parameters of the x-ray signals. The CWT performs a correlation of the kernel wavelet with the input signal. The CWT is defined in Equation (8).

$$WT(a,b) = \frac{1}{\sqrt{a}} \int f(t).\psi^*\left(\frac{t-b}{a}\right) dt \qquad (8)$$

In WISE a modified wavelet kernel is introduced to closely match the shape of the radiographic signals. This kernel includes several parameters that are optimized to maximize the correlation between the radiographic signal and the wavelet kernel. This algorithm allows better correlation, and consequently improves parameter estimation.

IV. Results

The performance of the thresholding technique applied to the DWT, DFT, and DCT coefficients is shown in Figure 4. This figure shows the relative energy compression ratio performance in radiographic experimental data. Since the DWT kernel resembles the x-ray signals, it offers better data compression compared to the DCT and DFT.

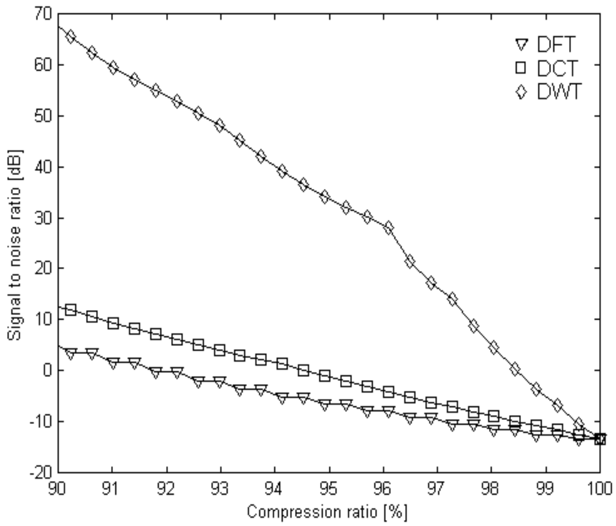
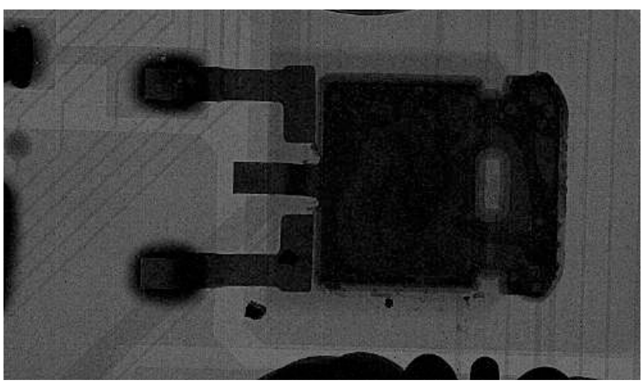


Figure 4 – Compression ratio performance with experimental data

The parameter estimation technique is applied to the noisy version of image in Figure 5a, while the reconstructed image is shown in Figure 5b.

a)



b)

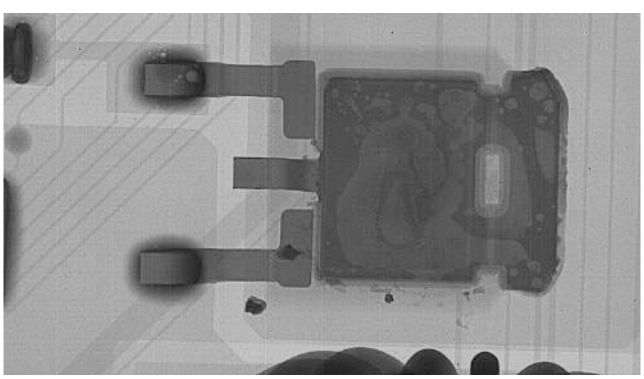


Figure 5 – Before and after of WISE: a) original image, b) reconstructed image.

The signal-to-noise ratio (SNR) of the signal is 6dB. In this example the parameter estimation technique is able to correctly isolate the x-ray signals of the radiographic image. A total of 35 coefficients are necessary to reconstruct the original signal with a SNR of 36dB.

V. Conclusions

This paper presents some of the different methods used in WISE to denoise and filter radiographic data. Thresholding techniques were analyzed using the DCT, DFT and DWT as a means to obtain frequency domain coefficients. The selection of the higher energy coefficient using an adaptive thresholding scheme reduces the amount of data to be stored and/or transmitted. In the experimental and simulated radiographic signals evaluated, the DWT outperformed the DFT and DCT. The high relative energy compression ratio obtained shows that these transform techniques are appropriate for the analysis and compression of radiographic signals.

The parameter estimation technique achieves even higher image improvement ratios. The CWT representation of the x-ray signal is used to design a window that isolates each single signal. As the number of signals increases the interference between them is higher, and consequently the performance of the estimation algorithm deteriorates. The same argument is applicable for noise interference. As SNR decreases, the algorithm will iterate to estimate the noise rather than the signals.

Thresholding techniques provide a high compression ratio, and are computationally simple to implement (e.g., various commercial chips are available to compute the DWT, DCT, and DFT). The parameter estimation demands a higher computational complexity, but with a higher compression ratio. The two techniques analyzed provide an elegant trade-off between implementation complexity and compression ratio.

VI. References

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